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Edge states and their transport in a quantum wire exposed to a non-homogeneous magnetic field

S. M. Badalyan† and F. M. Peeters‡

† Department of Radiophysics, Yerevan State University, 375049 Yerevan, Armenia
‡ Department of Physics, University of Antwerp (UIA), B-2610 Antwerpen, Belgium

Abstract. Edge states and their transport in a quantum wire exposed to a perpendicular non-homogeneous magnetic field are investigated. Systems are studied where the magnetic field exhibits a discontinuous jump in the transverse direction. The energy spectra and wave functions of these systems, the corresponding group velocities along the interface and the particle average positions normal to the interface are calculated. The resistance of the quantum wire is obtained both in the ballistic and in the diffusive regimes as a function of the Fermi energy and of the homogeneous background magnetic field.

Introduction

Investigations of semiconductor nanosystems is frequently connected with the use of magnetic fields. In the last several years a complex situation of nanosystems in a non-uniform magnetic field has attracted considerable interest [1]. Different experimental groups have succeeded in realizing such systems [2] by depositing patterned gates of superconducting or ferromagnetic materials on top of the heterostructure. An alternative approach to produce non-homogeneous magnetic fields is by varying the *topography* of an electron gas [3].

We investigate the magnetic edge states and their transport properties (in the ballistic and diffusive regimes) in nanosystems exposed to a normal inhomogeneous magnetic field. Structures are studied where the magnetic field changes its sign, strength, and both sign and strength at the magnetic interface. Such a system was recently realized experimentally [4] by depositing a ferromagnetic stripe on top of the electron gas and by applying a background magnetic field normal to the electron gas. Varying the background field results in all the above situations.

1. Approach

Consider a one-dimensional electron channel along the y-direction formed by the parabolic confining potential $V(x)$ and exposed to a normal non-homogeneous magnetic field $B_z(x) = B_1$ and $B_z(x) = -B_2$ respectively on the left and the right hand side of the magnetic interface at $x = 0$. This system is placed in a homogeneous background magnetic field $B_z(x) = B_b$. In any finite region along the x-direction where the magnetic field is uniform, the system is described by the single particle Hamiltonian $H = (\vec{p} + e/c\vec{A})^2/2m^* + V(x)$ where m^* is the particle effective mass, $V(x) = m^*\omega_0^2x^2/2$, ω_0 is the confining potential strength. We choose for the vector potential the Landau gauge $\vec{A} = (0, Bx, 0)$ and Schrödinger equation can be separated with the ansatz $\Psi(x, y) = e^{iky}\psi(x)$, where ψ is an eigenstate of the one-dimensional problem $(d^2/dx^2 + \nu + 1/2 - (x - X(k))^2/4)\psi(x - X(k)) = 0$. Here we introduce the following notations: $\nu + 1/2 = (\varepsilon - \hbar^2k^2/2m_B)/\hbar\omega^*$ is the particle transverse energy in units of $\omega^* = \sqrt{\omega_B^2 + \omega_0^2}$, ω_B is the cyclotron frequency,

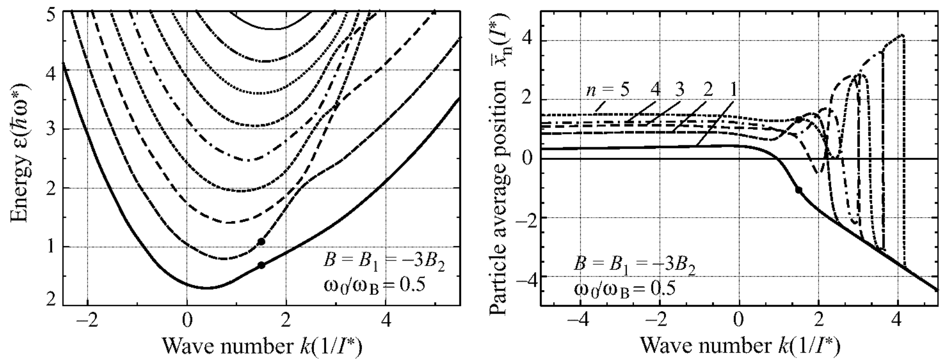

Fig. 1. The energy spectrum for the 8 lowest bands (left figure) and the particle average position corresponding to the 5 lowest energy bands (rigt figure).

ε and k are the energy and the momentum. The coordinate of the center of orbital rotation is $X(k) = kl^*\omega_B/\omega^*$ in units of the length scale $l^* = \sqrt{\hbar/(m^*\omega^*)}$. In the longitudinal direction the electron acquires a new field dependent mass $m_B = m^*\omega^{*2}/\omega_0^2$. The solutions of the above equation are the parabolic cylindrical functions $D_\nu(x)$.

In the non-homogeneous magnetic field case ν, X are different on the left and right hand side of the magnetic interface and we construct the wave function as $\psi_{\nu_1,\nu_2}(x, X_1, X_2) = D_{\nu_1}(\sqrt{2}(X_1(k) - x))$, if $x < 0$ and $\psi_{\nu_1,\nu_2}(x, X_1, X_2) = D_{\nu_2}(\sqrt{2}(x - X_2(k)))$, if $x > 0$. Indices 1, 2 refer to the values of quantities for which $\omega^* = \sqrt{\omega_B^2 + \omega_0^2}$ is taken with $B = B_1, B_2$, respectively. Matching of this wave function and its derivative at $x = 0$ leads to the dispersion equation $d\ln(D_{\nu_1}(x - X_1(k))/dx|_{x=-0} = d\ln(D_{\nu_2}(-x + X_2(k))/dx|_{x=+0}$. By solving this equation we obtain the energy and wave functions of the magnetic edge states, which are the solution of the one-dimensional problem with the effective potential $V_{eff}(x,k) = m\omega_1^{*2}(x - X_1(k))^2/2 + \hbar^2k^2/2m_{B_1}$, if $x < 0$ and $V_{eff}(x,k) = m\omega_2^{*2}(x - X_2(k))^2/2 + \hbar^2k^2/2m_{B_2}$, if $x > 0$. The shape of $V_{eff}(x,k)$ depends strongly on the sign of k and on the magnetic field profile.

2. Spectrum

For brevity here we restrict ourselves by consideration only asymmetric system: $B_1 \neq B_2$, $sign(B_1/B_2) = -1$. In this case $V_{eff}(x,k)$ exhibits a pronounced asymmetry both as a function of k and x. For negative values of k, $V_{eff}(x,k)$ is a triangular-like asymmetric well with a minimum of $\hbar^2k^2/2m^*$ at $x = 0$. For positive values of k, $V_{eff}(x,k)$ is a double well with different minima $\hbar^2k^2/2m_{B_1}$ and $\hbar^2k^2/2m_{B_2}$ at the positions $x = +X_1(k)$ and $x = -X_2(k)$, respectively. The triangular like barrier between the wells has again the height $\hbar^2k^2/2m^*$ at $x = 0$. Thus the confining potential together with the non-homogeneous magnetic field induces three effective masses (m^* for negative and m_{B_1}, m_{B_2} for positive values of k) in the system. The spectrum consists of alternating symmetrical and anti-symmetrical terms and is described by a discrete quantum number $n = 0, 1, 2, \ldots$ and the momentum k along the wire (Fig. 1). For negative values of k, the spectrum corresponds to snake orbits with free-like motion and with mass m^* along the y-direction. These states are effectively localized in the vicinity of the magnetic interface in the region where the magnetic field is smaller. The group velocity is approximately linear and the particle average

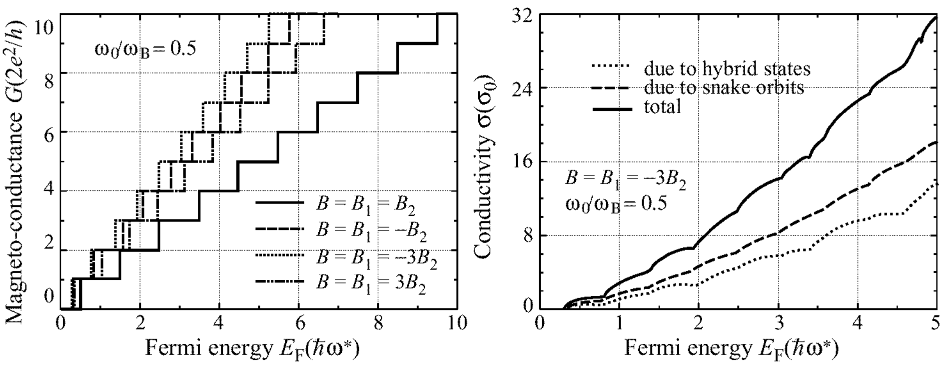

Fig. 2. The Fermi energy dependence of the conductance in the ballistic regime (left figure) and of the conductivity, in units of $\sigma_0 = e^2\tau/(\pi m^* l^*)$, in the diffusive regime (right figure).

position $\overline{x}_n$ is approximately independent of the wave number (Fig. 1). For positive k the spectrum characterizes the hybrid states. For some positive value of k the group velocity v_n and the particle average position $\overline{x}_n$ start to oscillate as a function of the wave number and the particle tunnels periodically from the left to the right side of the quantum wire and vice versa. At $k \to +\infty$ all states tend to be localized in the region where the magnetic field is large and the well of the effective potential is lower.

3. Transport

We calculate the zero temperature two terminal magneto-conductance for a perfect conductor using the Büttiker formula [5]. From Fig. 2 it is seen that the conductance, in the ballistic regime for different magnetic field profiles, exhibits stepwise variations as a function of the Fermi energy. For a given energy and confining potential strength, the conductance in the non-homogeneous magnetic field is nearly twice that for the homogeneous field case. The conductance decreases when going from the profile $B_1 = -3B_2$ to the profiles $B_1 = -B_2$, and $B_1 = +3B_2$.

The conductivity in the diffusive regime is calculated in the relaxation time approximation. We obtain $\sigma_{1D} = 2e^2/h\ \tau(E_F) \sum_n |v_n(k)|_{\varepsilon=E_F}$ in the zero temperature limit, τ is the momentum relaxation time. For the profiles $B_1 = -3B_2$ (see Fig. 2) and $B_1 = +3B_2$ the conductivity due to states with negative velocities (dashed curves) is larger than that due to states with positive velocities (dotted curves). In the case when the magnetic field changes its sign, the states with negative velocities are the snake states, which are always faster than the states with positive velocities which are related to the hybrid states. In the case of $B_1 = +3B_2$ all the states are hybrid states, however, the contribution to the conductivity of the states with negative velocities is larger because these states have the small mass m_B, and large velocity v_n. For both $v_n > 0$ and $v_n < 0$ parts, the conductivity has an oscillating structure as a function of the Fermi energy which is due to a divergence of the density of states at the bottom of the $\varepsilon_n(k)$ band. However, the contributions due to states with $v_n > 0$ exhibit an additional structure related to the oscillations of the group velocity as a function of k. This structure is more pronounced in the case of $B_1 = -3B_2$, the conductivity has additional distinct minima that reflect the tunneling effect discussed above.

The magneto-resistance in the ballistic regime exhibits stepwise variations as a function of the background magnetic field B_b. In the diffusive regime the resistance exhibits small

peaks as a function of B_b that are associated with the magnetic depopulation effect and that are on top of a positive magneto-resistance background, which increases with B_b.

4. Summary

We developed a theory for the non-homogeneous magnetic field induced edge states and their transport in a quantum wire. We calculated rigorously the spectrum of these systems, the corresponding group velocities along the magnetic interface and the particle average position normal to the magnetic interface. Exploiting these results, we calculated the conductance and the conductivity of the quantum wire in the ballistic and diffusive regimes.

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